whence

$$E + e < E_0 + e_0,$$

i.e., the total kinetic energy carried out by both the diverging jets is less than that brought in by the approaching jets by a value that is proportional to the unshaded area in Fig. 2b.

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COMPARISON OF ONE-DIMENSIONAL MODELS OF FLOWS IN BRANCHED CHANNELS WITH EXPERIMENTAL DATA

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One-dimensional modeling is presently the most popular approach to the description of gasdynamic flows in complicated systems containing a large number of tubes or channels coupled with each other. The so-called problem of the decay of an arbitrary discontinuity at a junction acquires an important role in the investigation of the general properties of generalized solutions of one-dimensional equations of gasdynamics in branched systems of channels. This problem has been investigated theoretically in sufficient completeness in a number of reports for the cases of couplings of two and three channels (jumps in cross section [1, 2], local resistance [3], a perforated barrier [4], branched channels with parallel axes [5, 6], and an arbitrary tee [7, 8]), and various self-similar solutions have been constructed. To obtain a complete picture, however, theoretical results must be compared with experimental data, which has been done so far only for certain particular cases of local resistances in two coupled channels. In the present work such a comparison is made for a plane tee formed by the main channel and a side opening of the same width.

We consider one particular case of the decay of a discontinuity, when a shock wave travels through quiescent gas to the branching section. Experiments of this type have been described sufficiently widely in the literature. As a result of the decay of the initial shock front a rarefaction wave is reflected in the main channel 1, while in the straight (channel 2) and side (channel 3) branches shock waves travel, behind which contact discontinuities follow. The self-similar flow pattern obtained in the one-dimensional model for this typical configuration is shown in the form of an x, t wave diagram in Fig. 1, where $x < 0$ corresponds to the main channel at the entrance to the tee, while $x > 0$ corresponds to one of the branches at the exit from the tee, R_1 is the reflected centered rarefaction wave, S_2 and

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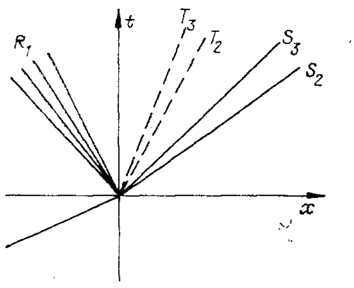

Fig. 1

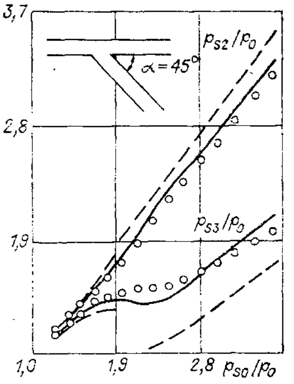

Fig. 2

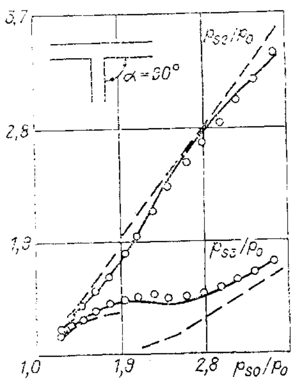

Fig. 3

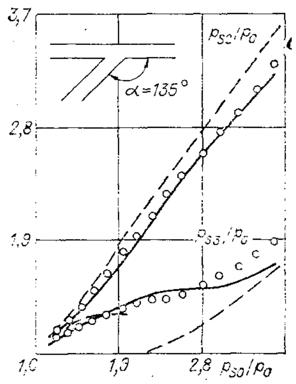

Fig. 4

S_3 are the shock waves which have passed through the branch, T_2 and T_3 are the contact discontinuities following them, and the t axis corresponds to the junction. With sufficiently strong initial shock waves, configurations are also possible with a deflected shock wave propagating in one of the branches upstream and located between the contact discontinuity and the junction.

The described decay of a discontinuity was calculated for several values of the angle α at which the side branch leaves the main channel. We used a standard subprogram for the numerical solution of the problem of the decay of an arbitrary discontinuity in a tee, created on the basis of the procedure developed in [7, 8], in accordance with which the conditions of coupling at the junction are written in the form

$$u_k = u_1 a(M_1, M_k), \quad p_k = p_1 a(M_1, M_k)\frac{M_1^2}{\beta_{1k} M_k^2},$$

$$\frac{M_1}{\beta_{1k} M_k}\left(\frac{1+\frac{\gamma-1}{2}M_k^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = \left\{\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_1^2[1+\xi_{1k}(\beta_{1k})]}\right\}^{\frac{\gamma}{\gamma-1}},$$

where

$$k = 2, 3; \quad \beta_{1k} = A_k/A_{1k}; \quad A_{12} + A_{13} = A_1; \quad a(M_1, M_k) = \frac{M_k}{M_1}\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_k^2}\right)^{1/2}; \; p_i, u_i, M_i$$

are the pressure, velocity, and Mach number in the channel; $i = 1, 2, 3$ in the zones of constant flow adjacent to the junction (see Fig. 1); γ is the adiabatic index; $A_1 = A_2 = A_3$ are the cross-sectional areas; $\xi_{1k}(\beta_{1k})$ is the local drag coefficient, calculated from an equation of the type

$$\xi_{1k}(\beta_{1k}) = \zeta_{1k}(\alpha_{1k})\left(1 + 1/\beta_{1k}^2 - 2\cos\alpha_{1k}/\beta_{1k}\right),$$

in which α_{1k} is the angle of stream deflection, while the factor ζ_{1k} was chosen for each branch in accordance with the recommendations on determining the local drag coefficients of plane tees in [9].

The results of a comparison of numerical results with experimental data for $\gamma = 1.4$ with $\alpha = 45$, 90, and 135°, are presented in Figs. 2-4, respectively. The graphs show the relative pressure behind shock waves which have passed through a branch (p_{S2}/p_0 and p_{S3}/p_0) as functions of the relative pressure behind the initial shock wave (p_{S0}/p_0): The solid curves are experiment [10], the dashed curves are the calculation by the procedure of [10], and the circles are the calculation by the procedure of [7, 8] of the decay of the discontinuity.

In conclusion, we note that this report is a kind of summing up of research on the one-dimensional theory of the decay of a discontinuity in a branching channel, and the agreement with experiment which was obtained confirms not only the legitimacy of the one-dimensional approach but also the possibility of using the local drag coefficients known for an incompressible fluid in practical calculations of nonsteady flows of a compressible gas in complicated systems of channels.

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